Mutual Transformations of Arbitrary Ternary Qubit Trees by Clifford Gates

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Editors: *Stefan K. Kolev & Danko D. Georgiev*

Article history: Submitted on September 3, 2026; Accepted on October 10, 2026; Published on October 11, 2026.

It is shown that ternary qubit trees with the same number of nodes can be transformed by the naturally defined sequence of Clifford gates into each other or into standard representation as 1-dimensional chain corresponding to Jordan–Wigner transform.
Quanta 2026; 15: 24–30.

1 Introduction

The *ternary qubit tree* [1,2] is a visual representation of Clifford algebra generators (or Majorana operators) as a multi-qubit Pauli operators [3,4]. It may be considered as a generalization of Jordan–Wigner transform [5] from a set of such operators naturally defined for 1-dimensional chain. The particular case of full ternary tree was suggested for optimal fermion-to-qubit mapping in quantum computations [6] and similar idea can be applied to more general ternary trees [7,8].

Qubit trees provide a method to construct set of operators with necessary commutation properties, however for some applications it would be desirable also to provide a map for conversion to standard Jordan–Wigner transform. The algorithm for representation of such map as a sequence of so-called stabilizer or Clifford gates [9–11] is one of the main results that is presented in this work.

The definition of ternary qubit trees and relation with Jordan–Wigner transform are recollected in Section 2. The sequence of transformations of such trees producing necessary stabilizer circuit is discussed in Section 3. Possible applications are briefly outlined in Section 4.

2 Clifford algebras and qubit trees

Let us recollect the definition of Clifford algebra $\boldsymbol{Cl}(n)$ generated by n elements with properties

$$\boldsymbol{e}_j\boldsymbol{e}_k + \boldsymbol{e}_k\boldsymbol{e}_j = 2\delta_{jk}\hat{\mathbb{1}}, \quad j,k = 1,\ldots,n, \tag{1}$$

where $\hat{\mathbb{1}}$ is unit of the algebra [3]. Often the right side of Eq. (1) includes a minus sign [2,3], but for complex Clifford algebra considered here such change is not essential due to possibility to multiply generators on imaginary unit. The Clifford algebras defined here should not be confused with Clifford gates also used in the present work. The generators Eq. (1) can be expressed using a tensor product of Pauli matrices

$$\hat{\sigma}^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\sigma}^y = \begin{pmatrix} 0 & -\imath \\ \imath & 0 \end{pmatrix}, \quad \hat{\sigma}^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2}$$

It should be noted that throughout the text, x, y, z are superscripts and not powers. We are reserving the subscript position for numbering the individual qubits in our trees.

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For even $n = 2m$ such representation corresponds to Jordan–Wigner transform [3, 5]

$$\begin{aligned} \boldsymbol{e}_{2k-1} &= \underbrace{\hat{\sigma}^z \otimes \cdots \otimes \hat{\sigma}^z}_{k-1} \otimes \hat{\sigma}^x \otimes \underbrace{\hat{\mathbb{1}} \otimes \cdots \otimes \hat{\mathbb{1}}}_{m-k} \\ \boldsymbol{e}_{2k} &= \underbrace{\hat{\sigma}^z \otimes \cdots \otimes \hat{\sigma}^z}_{k-1} \otimes \hat{\sigma}^y \otimes \underbrace{\hat{\mathbb{1}} \otimes \cdots \otimes \hat{\mathbb{1}}}_{m-k} \end{aligned}, \qquad k = 1, \ldots, m.$$

It can be rewritten

$$\boldsymbol{e}_{2k-1} = \Big(\prod_{j=1}^{k-1} \hat{\sigma}_j^z\Big)\hat{\sigma}_k^x, \qquad \boldsymbol{e}_{2k} = \Big(\prod_{j=1}^{k-1} \hat{\sigma}_j^z\Big)\hat{\sigma}_k^y, \qquad k = 1, \ldots, m. \tag{3}$$

using formal notation

$$\hat{\sigma}_k^\nu = \underbrace{\hat{\mathbb{1}} \otimes \cdots \otimes \hat{\mathbb{1}}}_{k-1} \otimes \hat{\sigma}^\nu \otimes \underbrace{\hat{\mathbb{1}} \otimes \cdots \otimes \hat{\mathbb{1}}}_{m-k}, \quad k = 1, \ldots, m, \quad \nu = x, y, z. \tag{4}$$

corresponding to the action of Pauli matrix on a qubit with index k.

Due to Eq. (1) the basis of Clifford algebra is 2^n different products of generators $\boldsymbol{e}_j$ and for $n = 2m$ due to Eq. (3) it coincides with $2^{2m} = 4^m$ different tensor products of m Pauli matrices. The Eq. (3) is not the only representation of elements with property Eq. (1) as a tensor product of Pauli matrices.

Let us recollect the big class of alternative representations with so-called ternary qubit trees [2, 6, 8]. Such trees have m qubit nodes joined with three children by directed links marked by indexes x, y, z and $2m + 1$ terminal nodes. An example of full ternary tree of such kind is represented on Figure 1, but arbitrary tree with up to three children nodes could be augmented to such ternary tree by attaching missing terminal nodes. An example of such ternary tree constructed from a binary one is shown on Figure 2.

Let us now associate $\hat{\sigma}_k^\nu$ with a link from qubit k with a label ν. Then each path P from a root to a terminal node corresponds to product of single-qubit Pauli operators Eq. (4) along this path according to the indices of the qubit nodes and the labels assigned to the outgoing arrows

$$\boldsymbol{e}_P = \prod_P \hat{\sigma}_{k(P)}^{\nu(P)}. \tag{5}$$

Any two such products $\boldsymbol{e}_P$ and $\boldsymbol{e}_{P'}$ include only one Pauli matrix with the same k and different $\nu \neq \nu'$. Here k is an index of the last qubit in the common part of paths P and P' before forking. Thus, any two unequal products Eq. (5)

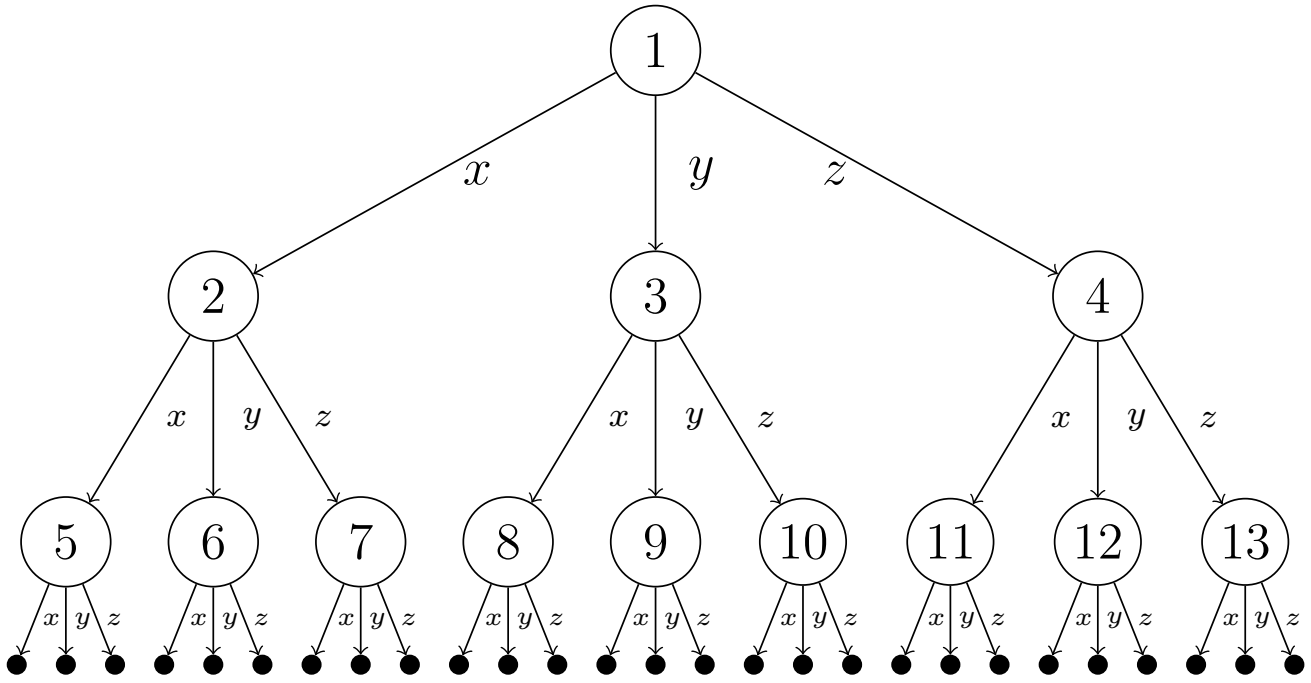

Figure 1: *Full ternary qubit tree.*

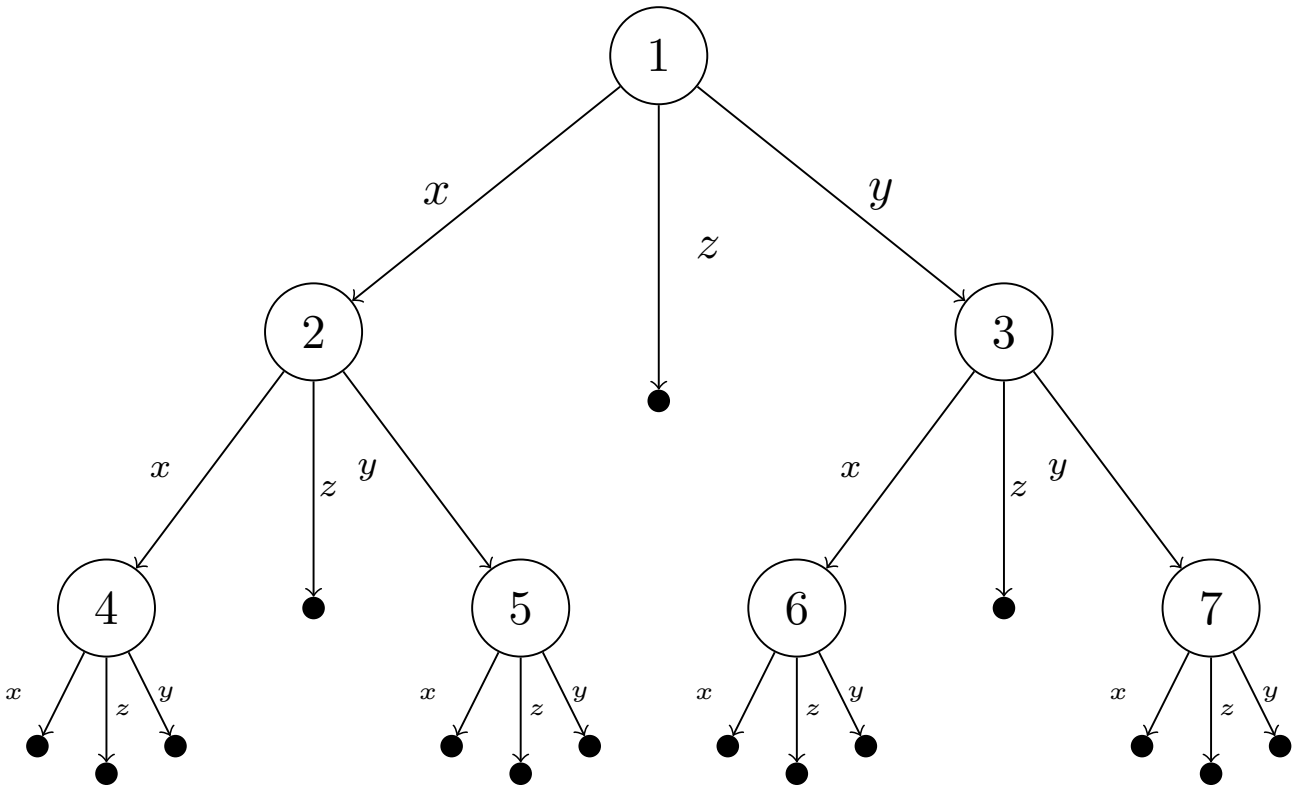

Figure 2: *Binary qubit tree augmented to ternary tree.*

anticommute and satisfy 'nontrivial part' $\boldsymbol{e}_j\boldsymbol{e}_k = -\boldsymbol{e}_k\boldsymbol{e}_j$, $j \neq k$ of Eq. (1). The 'trivial part' $\boldsymbol{e}_j^2 = \hat{\mathbb{1}}$ is also obviously satisfied.

For example, the full ternary tree on Figure 1 corresponds to 27 products

$$\boldsymbol{e}_1 = \hat{\sigma}_1^x\hat{\sigma}_2^x\hat{\sigma}_5^x,\ \boldsymbol{e}_2 = \hat{\sigma}_1^x\hat{\sigma}_2^x\hat{\sigma}_5^y,\ \boldsymbol{e}_3 = \hat{\sigma}_1^x\hat{\sigma}_2^x\hat{\sigma}_5^z,\ \boldsymbol{e}_4 = \hat{\sigma}_1^x\hat{\sigma}_2^y\hat{\sigma}_6^x, \ldots,\ \boldsymbol{e}_{27} = \hat{\sigma}_1^z\hat{\sigma}_4^z\hat{\sigma}_{13}^z$$

and the binary tree on Figure 2 produces 15 generators

$$\boldsymbol{e}_1 = \hat{\sigma}_1^x\hat{\sigma}_2^x\hat{\sigma}_4^x,\ \boldsymbol{e}_2 = \hat{\sigma}_1^x\hat{\sigma}_2^x\hat{\sigma}_4^z,\ \boldsymbol{e}_3 = \hat{\sigma}_1^x\hat{\sigma}_2^x\hat{\sigma}_4^y,\ \boldsymbol{e}_4 = \hat{\sigma}_1^x\hat{\sigma}_2^z, \ldots,\ \boldsymbol{e}_{15} = \hat{\sigma}_1^y\hat{\sigma}_3^y\hat{\sigma}_7^y.$$

There are $2m+1$ such products corresponding to the number of terminal nodes, but an arbitrary one can be dropped, because up to unessential "phase" any such $\boldsymbol{e}_P$ can be represented as a product of other $2m$ terms [2]. For the usual Jordan–Wigner transform Eq. (3) the extra term would correspond to

$$\boldsymbol{e}_{2m+1} = \prod_{j=1}^{k} \hat{\sigma}_j^z. \tag{6}$$

3 Transformations between representations

For any unitary operator $\hat{U}$ *adjoint action* (*conjugation* by $\hat{U}$) produces a new set of generators

$$\mathrm{Ad}_{\hat{U}} : \boldsymbol{e}_j \mapsto \hat{U}\boldsymbol{e}_j\hat{U}^{-1} = \hat{U}\boldsymbol{e}_j\hat{U}^{\dagger}, \quad j = 1, \ldots, n \tag{7}$$

satisfying the necessary commutation relations Eq. (1). For $\hat{U}$ represented by so-called *quantum stabiliser circuits* composed from product of Clifford gates [9, 10] Eq. (7) represents transformation between 4^k different products of $\hat{\sigma}_k^\nu$ Eq. (4). Such a property was discussed earlier for application to construction of new quantum gates effectively modelled by classical computer [12].

Let us recollect the construction of Clifford (stabilizer) gates [9, 10]. For a single qubit the group transforming $\hat{\sigma}^\nu$ into each other by conjugation may be generated by Pauli matrices $\hat{\sigma}^\nu$ together with Hadamard matrix $\hat{H}$ and 'phase gate' $\hat{S}$

$$\hat{H} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \hat{S} = \begin{pmatrix} 1 & 0 \\ 0 & \imath \end{pmatrix}. \tag{8}$$

Conjugation by Pauli matrices can only change a sign and be neglected further, but $\hat{H}$ and $\hat{S}$ exchanges $\hat{\sigma}^x \leftrightarrow \hat{\sigma}^z$ and $\hat{\sigma}^x \leftrightarrow \hat{\sigma}^y$ respectively.

For two qubits together with $\hat{H}_j$ and $\hat{S}_j$, $j = 1, 2$ acting on any of them it is necessary to include two-qubit gate Controlled-NOT ($\widehat{CX}$) or Controlled-Z ($\widehat{CZ}$) to generate the whole group of stabilizer transformations

$$\widehat{CX} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \widehat{CZ} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{9}$$

For arbitrary number of qubits compositions of gates $\hat{H}_j$, $\hat{S}_j$ acting on arbitrary qubit j together with $\widehat{CX}_{jk}$ or $\widehat{CZ}_{jk}$ acting on arbitrary pair of qubits (j, k) generate the whole group of transformations preserving the products $\hat{\sigma}^\nu_k$ (and the basis of Clifford algebra).

The exchange between arbitrary pair of qubits also may be decomposed on such operators, but for convenience it can be included as a separate gate. The operator $\widehat{CZ}_{jk}$ is used further due to convenience for some transformations of qubit trees, symmetry of two indexes and commutativity for different qubits.

Conjugation by $\widehat{CZ}_{12}$ for all possible tensor products of Pauli (and identity) matrices are collected in the table below

$\hat{\mathbb{1}} \otimes \hat{\mathbb{1}} \to \hat{\mathbb{1}} \otimes \hat{\mathbb{1}}$	$\hat{\sigma}^x \otimes \hat{\mathbb{1}} \to \hat{\sigma}^x \otimes \hat{\sigma}^z$	$\hat{\sigma}^y \otimes \hat{\mathbb{1}} \to \hat{\sigma}^y \otimes \hat{\sigma}^z$	$\hat{\sigma}^z \otimes \hat{\mathbb{1}} \to \hat{\sigma}^z \otimes \hat{\mathbb{1}}$
$\hat{\mathbb{1}} \otimes \hat{\sigma}^x \to \hat{\sigma}^z \otimes \hat{\sigma}^x$	$\hat{\sigma}^x \otimes \hat{\sigma}^x \to \hat{\sigma}^y \otimes \hat{\sigma}^y$	$\hat{\sigma}^y \otimes \hat{\sigma}^x \to \hat{\sigma}^x \otimes \hat{\sigma}^y$	$\hat{\sigma}^z \otimes \hat{\sigma}^x \to \hat{\mathbb{1}} \otimes \hat{\sigma}^x$
$\hat{\mathbb{1}} \otimes \hat{\sigma}^y \to \hat{\sigma}^z \otimes \hat{\sigma}^y$	$\hat{\sigma}^x \otimes \hat{\sigma}^y \to \hat{\sigma}^y \otimes \hat{\sigma}^x$	$\hat{\sigma}^y \otimes \hat{\sigma}^y \to \hat{\sigma}^x \otimes \hat{\sigma}^x$	$\hat{\sigma}^z \otimes \hat{\sigma}^y \to \hat{\mathbb{1}} \otimes \hat{\sigma}^y$
$\hat{\mathbb{1}} \otimes \hat{\sigma}^z \to \hat{\mathbb{1}} \otimes \hat{\sigma}^z$	$\hat{\sigma}^x \otimes \hat{\sigma}^z \to \hat{\sigma}^x \otimes \hat{\mathbb{1}}$	$\hat{\sigma}^y \otimes \hat{\sigma}^z \to \hat{\sigma}^y \otimes \hat{\mathbb{1}}$	$\hat{\sigma}^z \otimes \hat{\sigma}^z \to \hat{\sigma}^z \otimes \hat{\sigma}^z$

(10)

Let us consider the action of stabilizer gates on the qubit trees. The one-qubit gates such as $\hat{H}_j$, $\hat{S}_j$ and products of such operators map one qubit tree into another and simply exchange labels x, y, z of links attached to qubit j. For system with two qubits any set of generators may be represented as a ternary tree and so two qubit-gates $\widehat{CZ}$ also corresponds to some transformation of the trees.

For three and more qubits two-qubit gates such as $\widehat{CZ}_{jk}$ may transform products representing $\boldsymbol{e}_j$ for some ternary qubit tree into collection that either represent another tree or not. For example let us consider standard six generators Eq. (1) for three qubits

$$\boldsymbol{e}_1 = \hat{\sigma}^x_1,\ \boldsymbol{e}_2 = \hat{\sigma}^y_1,\ \boldsymbol{e}_3 = \hat{\sigma}^z_1\hat{\sigma}^x_2,\ \boldsymbol{e}_4 = \hat{\sigma}^z_1\hat{\sigma}^y_2,\ \boldsymbol{e}_5 = \hat{\sigma}^z_1\hat{\sigma}^z_2\hat{\sigma}^x_3,\ \boldsymbol{e}_6 = \hat{\sigma}^z_1\hat{\sigma}^z_2\hat{\sigma}^y_3. \tag{11}$$

Conjugation by $\hat{U} = \widehat{CZ}_{1\,2}\,\widehat{CZ}_{2\,3}$ produces six generators mentioned in Ref. [8] with correct commutation relation Eq. (1), but without representation as a ternary qubit tree

$$\tilde{\boldsymbol{e}}_1 = \hat{\sigma}^x_1\hat{\sigma}^z_2,\ \tilde{\boldsymbol{e}}_2 = \hat{\sigma}^y_1\hat{\sigma}^z_2,\ \tilde{\boldsymbol{e}}_3 = \hat{\sigma}^x_2\hat{\sigma}^z_3,\ \tilde{\boldsymbol{e}}_4 = \hat{\sigma}^y_2\hat{\sigma}^z_3,\ \tilde{\boldsymbol{e}}_5 = \hat{\sigma}^z_1\hat{\sigma}^x_3,\ \tilde{\boldsymbol{e}}_6 = \hat{\sigma}^z_1\hat{\sigma}^y_3. \tag{12}$$

Anyway, $\widehat{CZ}_{jk}$ also may produce valid transformation of ternary tree and it is shown further, how to map any ternary tree T into standard (Jordan–Wigner) form corresponding to Eq. (3) using Eq. (7) with some operator denoted here as $\hat{U}^\dagger_T$ with the inverse transformation $\hat{U}_T$ mapping Eq. (3) into set of generators for T. Arbitrary tree T also can be converted into another T' by composition of two such transformations $\hat{U}^\dagger_T\hat{U}_{T'}$ with possible further optimization to reduce the number of gates.

The idea is to 'straighten all forks' in the tree. Let us consider the last fork on one of the branches originated from some qubit with index q_1. It is depicted on Figure 3*a*. It is supposed, that there are no more forks on the branches attached to the qubit q_1, but these branches can be either terminal or include an arbitrary number of qubit nodes. For certainty the branch attached by link with label x has amount of nodes less or equal than others.

Let us show that conjugation by gate $\widehat{CZ}_{q_1q_2}$ rejoins the closest qubit node with index q_1 to the link with the label y, see Figure 3*b*. Here we suppose that node q_2 has two terminal nodes attached by links x and y with some tree T_1 may

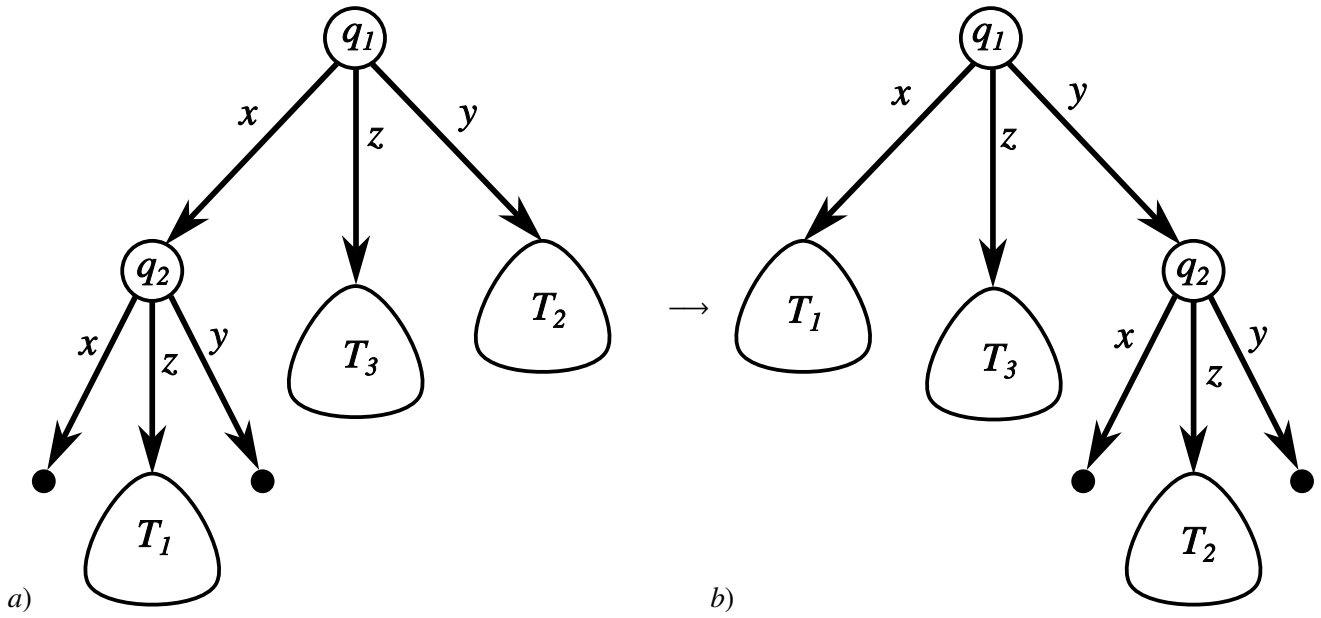

Figure 3: *Transformation of a qubit tree by* $\widehat{CZ}_{q_1 q_2}$.

be attached to link z. It is always possible to exchange labels in such a way by one-qubit transformation. Some trees T_2 and T_3 also may be attached to qubit node q_1 by links with labels y and z.

The scheme of tensor products of Pauli (and identity) matrices for generators corresponding to ternary qubit tree on Figure 3*a* is presented on the left-hand side of Eq. (13).

$$\begin{array}{ll} \cdots \hat{\sigma}^x \otimes \hat{\sigma}^x \otimes \hat{\mathbb{1}} \cdots \cdots & \cdots \hat{\sigma}^y \otimes \hat{\sigma}^y \otimes \hat{\mathbb{1}} \cdots \cdots \\ \cdots \hat{\sigma}^x \otimes \hat{\sigma}^y \otimes \hat{\mathbb{1}} \cdots \cdots & \cdots \hat{\sigma}^y \otimes \hat{\sigma}^x \otimes \hat{\mathbb{1}} \cdots \cdots \\ \cdots \hat{\sigma}^x \otimes \hat{\sigma}^z \otimes T_1 \cdots \cdots & \cdots \hat{\sigma}^x \otimes \hat{\mathbb{1}} \otimes T_1 \cdots \cdots \\ \cdots \hat{\sigma}^y \otimes \hat{\mathbb{1}} \otimes \cdots T_2 \cdots & \cdots \hat{\sigma}^y \otimes \hat{\sigma}^z \otimes \cdots T_2 \cdots \\ \cdots \hat{\sigma}^z \otimes \hat{\mathbb{1}} \otimes \cdots\cdots T_3 & \cdots \hat{\sigma}^z \otimes \hat{\mathbb{1}} \otimes \cdots\cdots T_3 \end{array} \qquad (13)$$

Here non-overlapping commuting terms T_1, T_2, T_3 correspond to subtrees on Figure 3. They could be arbitrary, yet only subtrees without forks are enough further. The positions corresponding to qubits q_1 and q_2 for simplicity are arranged sequentially in Eq. (13) and correspond to combinations

$$\hat{\sigma}^x \otimes \hat{\sigma}^x, \quad \hat{\sigma}^x \otimes \hat{\sigma}^y, \quad \hat{\sigma}^x \otimes \hat{\sigma}^z, \quad \hat{\sigma}^y \otimes \hat{\mathbb{1}}, \quad \hat{\sigma}^z \otimes \hat{\mathbb{1}}$$

mapping due to Eq. (10) into

$$\hat{\sigma}^y \otimes \hat{\sigma}^y, \quad \hat{\sigma}^y \otimes \hat{\sigma}^x, \quad \hat{\sigma}^x \otimes \hat{\mathbb{1}}, \quad \hat{\sigma}^y \otimes \hat{\sigma}^z, \quad \hat{\sigma}^z \otimes \hat{\mathbb{1}}$$

respectively. The result of such map is shown on the right-hand side of Eq. (13) and corresponds to the qubit tree depicted on Figure 3*b*.

Example of conversions from a triple fork to a linear chain is shown on Figure 4. Three branches with two nodes are attached to qubit q_1 by x, y and z links on Figure 4*a*. The gate $\widehat{CZ}_{1\,2}$ is used to move one qubit node from x to y branch, see Figure 4*b*. The gate $\widehat{CZ}_{1\,3}$ moves yet another qubit node from x to y and on Figure 4*c* now the x branch has a single terminal node. To continue the process it is necessary to exchange branches x and z by gate $\hat{H}_1$, see Figure 4*d*. Now it is possible to move qubit nodes q_6 and q_7 from x to y branch using gates $\widehat{CZ}_{1\,6}$ and $\widehat{CZ}_{1\,7}$, see Figure 4*e*. Now only y-branch has qubit nodes and gates $\hat{S}_1$ and $\hat{H}_1$ are used finally to exchange y and z links to produce a chain of z-linked nodes, see Figure 4*f*.

Consequent application of similar operations to all forks on a ternary qubit tree produces chain without any branches with nodes connected by z-links. It is only necessary to rearrange qubit indexes in ascending order to produce 'degenerate' qubit tree corresponding to a chain and standard (Jordan–Wigner) representation Eq. (3) of generators $\boldsymbol{e}_j$.

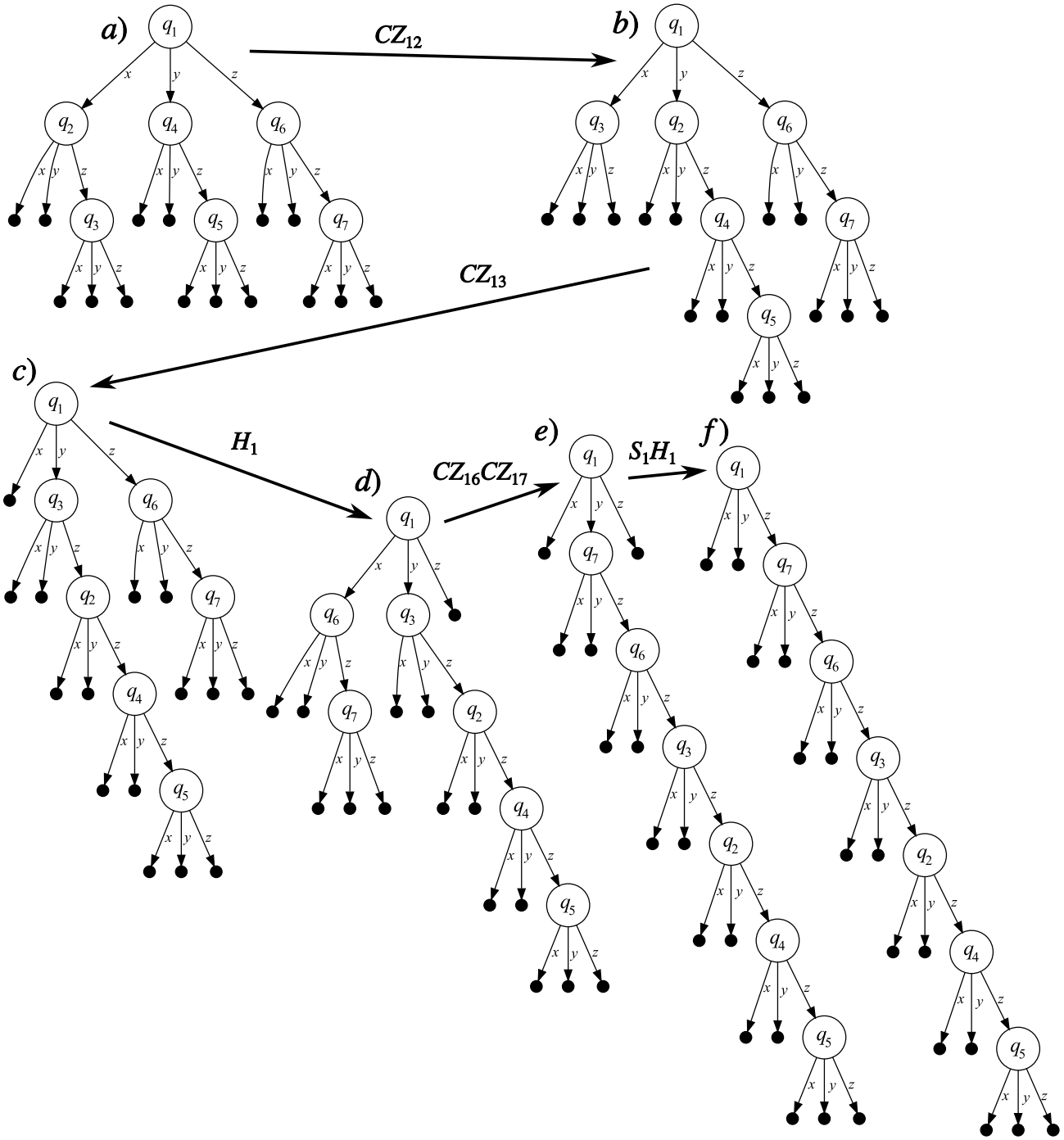

Figure 4: *Example of ternary qubit tree transformations by Clifford gates.*

4 Conclusion

In this work, I have constructed an adjoint map Eq. (7) between standard Jordan–Wigner transform Eq. (3) and representation by arbitrary ternary qubit tree Eq. (5) using the stabilizer circuit (the sequence of Clifford gates). The applications of such map can be numerous. The general idea of possible applications is similar with the one mentioned by Jozsa and Miyake [12] for the examples of quantum circuits 'intertwined by Clifford operations'. It was initially suggested for effective simulations of specific nonuniversal quantum circuits by classical computer, but some ideas recollected below can be useful for other applications.

The approach with conjugation by Clifford operations also looks more general, because not all set of generators expressed as Eq. (7) can be represented by ternary trees. However, such ideas appropriate for any set of generators represented using conjugation Eq. (7) and so may be applied to ternary trees as well due to the construction discussed in Section 3.

Adjoint transformation $\hat{G}' = \mathrm{Ad}_{\hat{U}}\hat{G}$ can be associated with transition from a quantum circuit $\hat{G}$ acting on a state $|\Psi\rangle$ to a new circuit $\hat{G}'$ for a quantum state $|\Psi'\rangle = \hat{U}|\Psi\rangle$ with trivial substitution

$$\hat{G}'|\Psi'\rangle = \hat{U}\hat{G}\hat{U}^\dagger|\Psi'\rangle = \hat{U}\hat{G}\hat{U}^\dagger\,\hat{U}|\Psi\rangle = \hat{U}\hat{G}|\Psi\rangle.$$

Any observable $\hat{O}$ used in the measurement $\langle\Psi|\hat{O}|\Psi\rangle$ for some states in the initial circuit would correspond to a new observable $\hat{O}' = \hat{U}\hat{O}\hat{U}^\dagger$. If such observables were expressed as an analytical function of generators $\boldsymbol{e}_k$, such as quadratic, polynomial or exponential, the new observables also can be expressed in the same way. Similar property is also valid for Hamiltonians $\hat{H}$ used for formal expression of quantum evolution

$$|\Psi(t)\rangle = \hat{G}(t)|\Psi_0\rangle = e^{-\imath\hat{H}t}|\Psi_0\rangle,$$

where for the modified quantum system or circuit, the Hamiltonian can be expressed as $\hat{H}' = \hat{U}\hat{H}\hat{U}^\dagger$. Here the general ideas of possible applications are only outlined and concrete examples will be presented elsewhere.

References

[1] A. Y. Vlasov. Ternary qubit trees. In: Conference "Quantum optics and related topics 2026", September 26–October 4, 2026, Online. Steklov Mathematical Institute of Russian Academy of Sciences, Moscow. https://www.mathnet.ru/eng/present51599.

[2] A. Y. Vlasov. Clifford algebras, spin groups and qubit trees. *Quanta* 2022; **11**:97–114. doi:10.12743/quanta.v11i1.199.

[3] J. E. Gilbert, M. A. M. Murray. Clifford Algebras and Dirac Operators in Harmonic Analysis. Cambridge University Press, Cambridge, 1991. doi:10.1017/cbo9780511611582.

[4] P. Lounesto. Clifford Algebras and Spinors. 2nd Edition. Cambridge University Press, Cambridge, 2009. doi:10.1017/CBO9780511526022.

[5] P. Jordan, E. P. Wigner. Über das Paulische Äquivalenzverbot. *Zeitschrift für Physik* 1928; **47**(9):631–651. doi:10.1007/BF01331938.

[6] Z. Jiang, A. Kalev, W. Mruczkiewicz, H. Neven. Optimal fermion-to-qubit mapping via ternary trees with applications to reduced quantum states learning. *Quantum* 2020; **4**:276. doi:10.22331/q-2020-06-04-276.

[7] J. Tilly, H. Chen, S. Cao, D. Picozzi, K. Setia, Y. Li, E. Grant, L. Wossnig, I. Rungger, G. H. Booth, J. Tennyson. The Variational Quantum Eigensolver: A review of methods and best practices. *Physics Reports* 2022; **986**:1–128. doi:10.1016/j.physrep.2022.08.003.

[8] A. Miller, Z. Zimborás, S. Knecht, S. Maniscalco, G. García-Pérez. Bonsai algorithm: grow your own fermion-to-qubit mappings. *PRX Quantum* 2023; **4**(3):030314. doi:10.1103/PRXQuantum.4.030314.

[9] D. Gottesman. Theory of fault-tolerant quantum computation. *Physical Review A* 1998; **57**(1):127–137. doi:10.1103/PhysRevA.57.127.

[10] M. A. Nielsen, I. L. Chuang. Quantum Computation and Quantum Information. Cambridge University Press, Cambridge, 2010. doi:10.1017/cbo9780511976667.

[11] G. Biswas. On classical simulation of quantum circuits composed of Clifford gates. *Quanta* 2024; **13**:20–27. doi:10.12743/quanta.v13i1.265.

[12] R. Jozsa, A. Miyake. Matchgates and classical simulation of quantum circuits. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 2008; **464**(2100):3089–3106. doi:10.1098/rspa.2008.0189.